SECOND TERM EXAMINATION

MATHEMATICS

(Class X)

(Arithmetic progression, Triangles, Introduction to trigonometry, Coordinate Geometry, Statistics)

Solutions

1) c) 10^{th} term

2) d) $\sqrt{a^2 + b^2}$

3) d) 3 + $4\sqrt{3}$

4) b) Median

5) d) 7/13

6) x = 1

7) $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}\right) = (0,0)$

$a + b + c = 0$

$a^3 + b^3 + c^3 = 3abc$

8) $\frac{2x}{x-3} = \frac{x+3}{x-3}$

$2x - x = 3$

$x = 3$

9) $tan3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$

$tan3x = \frac{1}{2} + \frac{1}{2}$

$tan3x = 1$

$3x = 45$

$x = 15°$

10) $\Delta ABC \sim \Delta DEF$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\frac{3}{DE} = \frac{2}{4} = \frac{2.5}{DF}$

$DF = 5cm$

$DE = 6cm$

$P.of\ \Delta DEF = 5 + 6 + 4 = 15cm$

11) $S_6 = 42$

$\frac{6}{2}[2a + 5d] = 42$

$2a + 5d = \frac{42}{3}$

$2a + 5d = 14$ (1)

$\frac{a_{10}}{a_{30}} = \frac{1}{3}$

$3[a + 9d] = a + 29d$

$3a + 27d = a + 29d$

$2a = 2d$

$a = d$ (Subs in (1))

7a = 14

a = 2

d = 2

a_{13} = a + 12d
= 2 + 24
= 26

12) Let O(x,y)
A(2,1) B(5, -8) C(2, -9)
OA = OB
$(x-2)^2 + (y-1)^2 \pm (x-5)^2 + (y+8)^2$
$x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 + 10x + 25 + y^2 + 16y + 6y$
$-4x - 2y + 5 = -10x + 16y + 89$
$6x - 18y = 84$
$x - 2y = 14$ (1)
OA = OC
$(x-2)^2 + (y-1)^2 = (x-2)^2 + (y+9)^2$
$y^2 - 2y + 1 = y^2 + 18y + 81$
$-20y = 80$
$y = -4$ (Subs in (1))
$x + 12 = 14$
$x = 2$
(2, – 4)

13) $\Delta AFG\ and\ \Delta DBG$
$\llcorner GAF = \llcorner BDG$ (90°)
$\llcorner AGF = \llcorner DBG$ [Corresponding angles]

by AA ~
$\Delta AGF \sim \Delta DBG$ (1)

$\Delta AGF\ and\ \Delta EFG$
$\llcorner AFG = \llcorner CEF$ (90°)

$\llcorner AFG = \llcorner ECF$ [corresponding]

by AA~

$\Delta AGF \sim \Delta EFC$ (2)

$\Delta DBG \sim \Delta EFC$

by CPCT

$$\frac{BD}{EF} = \frac{DG}{EC}$$

$$\frac{BD}{DE} = \frac{DE}{EC}$$

$$DE^2 = BD \times EC$$

14)

C.I	f
4.5 – 14.5	6
14.5 – 24.5	11
24.5 – 34.5	21
34.5 – 44.5	23
44.5 – 54.5	14
54.5 – 64.5	5

Modal class

l = 34.5 $f_1 = 23$

$f_0 = 21$ $f_2 = 14$ h = 10

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 34.5 + \left[\frac{23-21}{2\times23-21-14}\right] \times 16$$

$= 34.5 + \left[\frac{2}{46-35}\right] \times 10$

$= 34.5 + \frac{20}{9}$

$= 36.72$

15) sinθ + cosθ = $\sqrt{2}$sin (90 – θ)

sinθ + cosθ = $\sqrt{2}$cosθ

sinθ = $\sqrt{2}$cosθ – cosθ

$$\frac{sin\theta}{cos\theta} = \sqrt{2} - 1$$

$$tan\theta = \sqrt{2} - 1$$

$$cot\theta = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$cot\theta = \sqrt{2} + 1$$

16)

C.I	f	C.f
0 – 10	x	x
10 – 20	5	5 + x
20 – 30	9	14 + x
30 – 40	12	26 + x
40 – 50	y	26 + x + y
50 – 60	3	29 + x + y
60 – 70	2	31 + x + y
	40	

x + y = 9 (1)

l = 30

f = 12

C.f = 14 + x

$$32.5 = 30 + \frac{20-(14+x)}{12} \times 10$$

$$2.5 = \frac{(6-x)}{12} \times 10$$

$$\frac{2.5 \times 12}{10} = 6 - x$$

$$3 = 6 - x$$

$x = 3$ (Subs in (1))

y = 6

17) $\Delta PQR \ at \llcorner Q = 90°$

$PR^2 = PQ^2 + QR^2$ (1)

$\Delta PQS \ at \llcorner Q = 90°$

$$PS^2 = PQ^2 + QS^2$$

$$PQ^2 = PS^2 - QS^2$$

$$= PS^2 - \left(\frac{QR}{2}\right)^2$$

$$= PS^2 - \left(\frac{QR^2}{4}\right)$$

$$4PQ^2 = 4PS^2 - QR^2$$

$QR^2 = 4PS^2 - 4PQ^2$ (Subs in (1))

$$PR^2 = PQ^2 + 4PS^2 - 4PQ^2$$

$$PR^2 = 4PS^2 - 3PQ^2$$

18) $S_m = n$

$S_n = m$

$\frac{m}{2}[2a + (m-1)d] = n$

$2am + m(m-1)d = 2n$ (1)

$2an + n(n-1)d = 2m$ (2)

(1) – (2)

2a + (m + n – 1)d = – 2 (3)

$S_{m+n} = \frac{m+2}{2}[2a + (m+n-1)d]$

$= \frac{m+n}{2}(-2)$

$= -(m+2)$ (from (3))

19) $P\left[\frac{-10+(-2)}{2}, \frac{4+0}{2}\right]$

$P[-6,2]$

P divides CD in the ratio

K = 1

C (-9, -4) D (-4, y)

$(-6,2) = \left[\frac{K(-4)+1(-9)}{K+1}, \frac{K(y)+1(-4)}{K+1}\right]$

$-6 = \frac{-4K-9}{K+1}$ $2 = \frac{Ky-4}{K+1}$

$-6K - 6 = -4K - 9$

$-2K = -3$

$K = \frac{3}{2}$

$$2 = \frac{\frac{3}{2}y - 4}{\frac{3}{2} + 1}$$

$$2 = \frac{3y - 8}{2\left[\frac{5}{2}\right]}$$

$$10 = 3y - 8$$

$$y = 6$$

20) $\frac{cos\theta - sin\theta}{cos\theta + sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$$\frac{cos\theta\left[1 - \frac{sin\theta}{cos\theta}\right]}{cos\theta\left[1 + \frac{sin\theta}{cos\theta}\right]} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$\frac{1 - tan\theta}{1 + tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$tan\theta = \sqrt{3}$$

$$\theta = 60°$$

21) $\frac{\cos^2 20 + \sin^2 20}{\sec^2 50 - \tan^2 50} + 2[cosec^2 58 - \cot^2 58]$

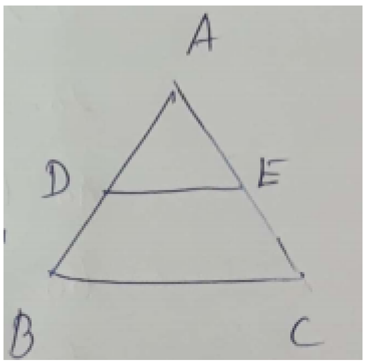
$$-4tan13.tan3(1) \times cot13.cot37$$

$$= \frac{1}{1} + 2(1) - 4(1)$$

$$= 1 + 2 - 4$$

$$= -1$$

22) Given:



23) $1+\sin^2\theta = 3sin\theta cos\theta$

$$\frac{1+\sin^2\theta}{\cos^2\theta} = \frac{3sin\theta cos\theta}{\cos^2\theta}$$

$$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 3\frac{sin\theta}{cos\theta}$$

$$\sec^2\theta + \tan^2\theta = 3tan\theta$$

$$1+\tan^2\theta + \tan^2\theta - 3tan\theta = 0$$

$$2\tan^2\theta - 3tan\theta + 1 = 0$$

$$(2tan\theta - 1)(tan\theta - 1) = 0$$

$$tan\theta = \frac{1}{2} \quad and \;\; tan\theta = 1$$

24)

C.I	f	Mid(x)	$d = \frac{x-A}{n}$	fd
0 – 20	17	10	-2	-34
20 – 40	f_1	30	-1	$-f_1$
40 – 60	32	50	0	0
60 – 80	f_2	70	1	f_2
80 – 100	19	90	2	38

$68 + f_1 + f_2 = 120$

$f_1 + f_2 = 52$ (1)

$$50 = 50 + \frac{\sum fd}{\sum f} \times n$$

$$0 = \frac{0 - f_1 + f_2}{120} \times 20$$

$$0 = 4 - f_1 + f_2$$

$f_1 - f_2 = 4$ (2)

Solve (1) *and* (2) *we get,*

$f_1 = 28,$

$f_2 = 24$